equivalent circuit. The phase difference of suitably paired response curves is given in Fig. 5, the phase error is seen to be below ±5 degrees in this experimental arrangement. At higher power levels additional phase errors may occur due to increase of the effective junction capacitance in a nonlinear capacitor. Further errors result from rectification and resulting change of bias and from heating of the junction.

Diodes having a cutoff frequency of 40 GHz (at −6 volts) were used in the experimental circuit. Attenuation is a function of phase[3] and with these diodes the maximum and minimum values of the attenuation differed by less than 1.8 dB over the whole phase and frequency range which agreed well with the theoretical expectations. However, for better diodes with a breakdown voltage of −24 volts and cutoff frequency of 160 GHz (at −6 volts) this difference in attenuation would be only 0.5 dB, which is acceptable for most applications.

The reflection factor at the input of the phase shifter was less than 20 percent. If better matching is required, substantially smaller reflection factors can be achieved by using end half sections.

Finally, it should be mentioned that feeding the bias voltage to the center conductor through helical lines proved to be very successful. The helical lines were made long enough to provide enough attenuation at the signal frequency, so that they appear to be terminated by their characteristic impedance. The main transmission line with a characteristic impedance of approximately 50 ohms is loaded only imperceptibly by the high impedance of the helical lines (2.3 kΩ). The rise-time of the bias voltage in this arrangement was less than 30 ns.

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Bandwidth Curves for Mumford's Maximally Flat Filter

In 1963, Mumford published design tables for a maximally flat symmetrical bandpass filter composed of six or fewer shorted quarterwave stubs separated by quarterwave lines of unit impedance.[1] The tables were extended to ten stubs in 1965.[2] These tables list the stub normalized admittances k_i for given values of n (the number of stubs) and a bandwidth-related parameter (10 log K_n) calculated from the k_i.

The bandwidth curves given here (Fig. 1) are supplementary design information which were computed from Mumford's formula for insertion loss

$$\frac{P_0}{P_L} = 1 + K_n \frac{(\cos\theta)^{2n}}{(\sin\theta)^2} \qquad (1)$$

where $\theta = 2\pi L/\lambda$, L is the length of a stub, and λ is the wavelength at center frequency in the passband. The curves serve a twofold purpose. First, they give the relative 3 dB bandwidth w as a function of (10 log K_n) and n. Here,

$$w = (f_2 - f_1)/f_0 = 2(f_0 - f_1)/f_0 \qquad (2)$$

where $(f_2 - f_1)$ is the 3 dB bandwidth and f_0 is the center frequency. Second, they permit the determination of attenuation L_s at frequency f_s for values of attenuation $L_s > 15$ dB. Thus, at the two edges of the band centered at f_0

$$w_s = |2(f_0 - f_s)/f_0| \qquad (3)$$

we have

$$L_s \approx 10 \log_{10} K_n - (10 \log_{10} K_n)_s \qquad (4)$$

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[1] W. W. Mumford, "An exact design technique for a type of maximally flat quarter-wave coupled band pass filter," *IEEE-PGMTT Nat'l Symp. Prog. and Digest*, pp. 57–61, May 1963.

[2] W. W. Mumford, "Tables of stub admittances for maximally flat filters using shorted quarter-wave stubs," *IEEE Trans. Microwave Theory and Techniques (Correspondence)*, vol. MTT-13, pp. 695–696, September 1965.

where the first term on the right side is the same as previously described, and the second term (with subscript s) is obtained from the bandwidth curves of Fig. 1 for any chosen (high-attenuation) bandwidth $w_s > w$.

The following example will explain the use of the bandwidth curves. Consider the design of a six-stub filter with a specified 3 dB bandwidth $w = 0.72$. (This value was chosen to allow comparison with the trial-design insertion-loss curve in Fig. 1 of Mumford,[1] which has the same relative 3 dB bandwidth.) First, enter Fig. 1 at $w = 0.72$ on the $(n=6)$ curve. We obtain 10 log $K_6 = 30.5$ which may then be used to find, by interpolation in Mumford's table, the normalized admittances k_i for a six-stub filter. (This calculation will be left to the interested reader.) Next, assuming the center frequency of the filter to be 900 MHz, as in Mumford's example, find the attenuation at 425 MHz. First, using (3), calculate $w_s = 2(900 - 425)/900 = 1.06$. Entering Fig. 1 again, we obtain (10 log $K_6)_s = 12.5$, then by (4), $L_s = 30.5 - 12.5 = 18$ dB, which checks closely with Mumford's Fig. 1.[1]

For values of attenuation near zero frequency (or near the center of the upper stopband), the attenuation L_s depends mainly on the value of K_n and the frequency, and is relatively independent of the number of stubs n. This is seen in Fig. 1 for large values of bandwidth approaching the upper limit $w_s = 2$. The reason for this characteristic of L_s is the well-known fact that, regardless of the value of n, there is only one pole of attenuation at zero frequency for the shorted quarterwave stub configuration of a bandpass filter. A useful formula for computing the attenuation near zero frequency is

$$L_s(w_s \to 2) \approx 10 \log K_n - 20 \log\left(1 - \frac{w_s}{2}\right) - 4 \text{ dB}. \qquad (5)$$

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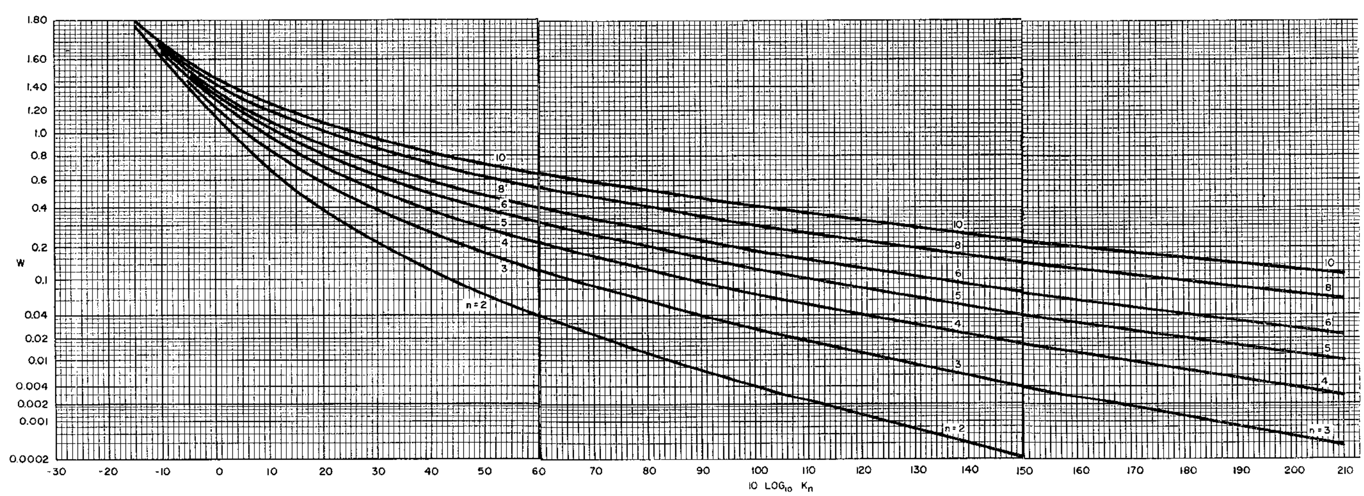

Fig. 1. Relative bandwidth w plotted against filter design parameter (10 $\log_{10} K_n$) for Mumford's maximally flat filter.